

# Research on Urban Distribution Route Optimization with Time-Varying Networks and Low-Carbon Emissions

Jing Wang<sup>1,\*</sup>, Hui Wang<sup>1</sup> and Qian Liu<sup>1</sup>

<sup>1</sup>School of Management, Wuhan University of Technology, Hongshan District, Wuhan, China

\*Corresponding author

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**Abstract:** In view of the increasing demand for urban distribution in traffic congestion environment, the calculation method of vehicle speed and travel time across time periods in time-varying network from the change law of urban traffic flow is put forward. Considering the influence of distance, time, speed, load and fuel factors on the cost, a mathematical model with the sum of fuel consumption cost, carbon emissions cost and fixed vehicle cost as the optimization objective is set up and the genetic algorithm is applied to solve the model. The comparison of the path optimization arrangements from the traditional TDVRP model with the total mileage minimization and carbon emissions minimization verify the effectiveness and feasibility of the model and algorithm in this paper.

## 1. Introduction

Based on urban traffic rules, this paper divides the distribution time window into multiple time periods, considering the impact of vehicle speed change on fuel consumption and carbon emissions, and combined with the current carbon trading mechanism to establish the optimization model including the cost of carbon emissions. The genetic algorithm is used to solve the distribution path with the lowest total cost, and the feasibility of the model is verified.

## 2. Model establishment

The distribution center has a group of homogenous fleets, which can complete the delivery tasks of multiple customers in the covered area according to the time and quantity requirements of customers in the delivery time window, and need to return to the distribution center after the completion of distribution. In the process of distribution, due to the influence of urban road traffic conditions, the traffic flow of the road section between different demand points is different in different time areas, and the speed of vehicle presents a time-varying characteristic, which also affects the fuel amounts and carbon emission of vehicles and has a direct impact on the distribution costs.

### 2.1 Assumption

(1) A single distribution center, the starting point and the end point of the vehicle are the distribution center; (2) The delivery vehicles have the same model and the same vehicle capacity. The load capacity of all vehicles must not exceed the limit load; (3) Each delivery point has a hard time window constraint and needs to be serviced within the time window; (4) Each delivery point can only be serviced by one car, and delivery can be completed only once; (5) The vehicle only delivers products to customers and does not provide pick-up service; (6) The distribution time window is divided into several time periods, and the vehicle speed remains the same on the route formed between any two distribution points; (7) The engine of the vehicle is turned off during the service of the customer, and there is no fuel consumption and no carbon emissions.

## 2.2 Parameters and variables

$G = (N, S)$ : distribution network;  
 $N = \{N_0 \cup N_c\}$ : node set,  $N_0$  represents distribution center,  $N_c$  represents customer set;  
 $S = \{(i, j) | \forall i \in N, j \in N, i \neq j\}$ : a collection of paths between two nodes;  
 $K = \{0, 1, 2, 3, \dots, k\}$ : vehicles collection;  
 $T = \{1, 2, 3, \dots, R\}$ : time periods collection;  
 $l_r$ : the length of the  $r$  time zone;  
 $t_r$ : the end time of the  $r$  time zone;  
 $d_{ij}$ : the distance from node  $i$  to node  $j$ ;  
 $v_{ij}^r$ : the average travel speed of vehicle from  $i$  to  $j$  in the  $r$  time zone;  
 $\tau_{i,jk}$ : the departure time of vehicle  $k$  from node  $i$  to node  $j$ ;  
 $e_{ij}$ : the arrival time of vehicle  $k$  from node  $i$  to node  $j$ ;  
 $c_d$ : the fixed cost of vehicle;  
 $c_f$ : cost per unit fuel consumption;  
 $c_e$ : unit carbon trading price;  
 $AE$ : carbon emission limit;  
 $m$ : coefficient of carbon emission conversion;  
 $q_i$ : the demand of the customer node  $i$ ;  
 $f_{i,jk}$ : the cargo weight when the vehicle  $k$  leaves the node  $i$ ;  
 $\mu$ : weight of vehicle;  
 $Q$ : maximum load of vehicle;  
 $\delta_1$ : parameters of engine module,  $\delta_1 = KNV$ ;  
 $\delta_2$ : parameters of speed module,  $\delta_2 = \gamma\beta$ ;  
 $\delta_3$ : parameters of weight module,  $\delta_3 = \gamma\alpha$ ;  
 $T_{ijk}^{mn}$ : the travel time when the vehicle  $k$  leaves in the  $m$  time zone and arrives in the  $n$  time zone;  
 $s_j$ : time for loading and unloading service at the customer node  $j$ ;  
 $[a_j, b_j]$ : the time window requirements of the customer node  $j$ ;  
the decision variables:  
 $x_{i,jk}$ : whether the path  $(i, j)$  is visited by the vehicle, 1 means yes, 0 means no;  
 $y_{i,jk}^{mn}$ : whether the departure time of vehicle  $k$  is in the  $m$  time zone and the arrival time is in the  $n$  time zone, 1 means yes, 0 means no;  
 $h_{ijk}^{mn}$ : whether the departure time of vehicle  $k$  is less than  $t_m$ , the arrival time is more than  $t_n$ , 1 means yes, 0 means n.  
 $u_{ijk}^m$ : whether the departure time of vehicle  $k$  is more than  $t_m$ , 1 means yes, 0 means no;  
 $w_{ijk}^m$ : whether the departure time of vehicle  $k$  is in the  $m$  time zone, 1 means yes, 0 means no.

## 2.3 Calculation of vehicle speed based on traffic flow

The federal highway function in the United States quantifies the relationship between traffic flow and vehicle speed, as shown in (1).  $v_0$  is the vehicle driving speed when the traffic flow on the road section is zero,  $q$  is the real-time traffic flow,  $c$  is the basic traffic capacity of the road section, and  $\varepsilon_1, \varepsilon_2$  is the model parameter. The relationship between vehicle speed and traffic flow is given

according to the type of road <sup>[1]</sup>, as shown in table 1.

$$v = \frac{v_0}{1 + \varepsilon_1 \left( \frac{q}{c} \right)^{\varepsilon_2}} \quad (1)$$

Table 1 Speed-Flow relation table of various types of various road in urban road network.

Road Type	Free Flow Speed $v_0$ (km/h)	Basic Capacity $c$ (pcu/h)
Main Road	60	1650
Collector Road	40	1200
Bypass	30	700

## 2.4 Calculation of travel time between customer points

In the changing road network, the change of traffic flow in a day can be divided into  $R$  periods according to its operation law, in other words  $T = T_1, T_2, \dots, T_R$ . The end time of the  $r_{th}$  time period is  $t_r$ , and the time span is  $l_r$ . Assuming the distance of arc  $(i, j)$  is  $d_{ij}$ , The diagram of the speed across time periods is shown in Figure 1. The departure time of the vehicle  $\tau_{ij}$  from the ground is within the  $m$  time period, and the end time of the time period is  $t_m$ .

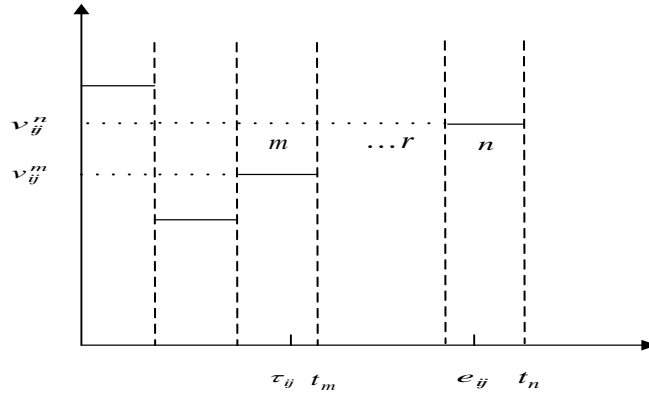


Figure 1 Vehicle Speed Diagram of Cross Time Periods from Node  $i$  to Node  $j$ .

$$\begin{cases} \begin{cases} t_{m-1} \leq \tau_{ij} \leq t_m \\ \sum_{m \in T \setminus \{R\}} u_{ijk}^m + x_{ijk} = \sum_{m \in T} m w_{ijk}^m \\ \sum_{m \in T \cup \{0\}} w_{ijk}^m = 1 \end{cases} & (2) \\ \begin{cases} (t_m - \tau_{ij}) v_{ij}^m + \sum_{r=m+1}^{r \leq n-1} l_r v_{ij}^r \leq d_{ij} < (t_m - \tau_{ij}) v_{ij}^m + \sum_{r=m+1}^{r \leq n} l_r v_{ij}^r \\ \sum_{m \in T \setminus \{R\}} h_{ij}^{mn} + m w_{ij}^m = \sum_{m \in T} n h_{ij}^{mn} \\ \sum_{m \in T \cup \{R+1\}, n \geq m} h_{ij}^{mn} = 1 \end{cases} & (3) \end{cases}$$

After determining the value of  $m$  and  $n$ , it can be known that the vehicle's travel distance in the time period  $m$  is  $(t_m - \tau_{ij}) v_{ij}^m$ , and the travel distance between time periods  $m$  and  $n$  is  $\sum_{r=m+1}^{r \leq n-1} l_r v_{ij}^r$ , so the cross-period travel time of the vehicle  $T_{ij}^{mn}$  from  $i$  to  $j$  can be calculated according to formula (4):

$$T_{ij}^{mn} = (t_{n-1} - \tau_{ij}) + \left[ d_{ij} - (t_m - \tau_{ij}) v_{ij}^m - \sum_{r=m+1}^{r \leq n-1} l_r v_{ij}^r \right] / v_{ij}^n \quad (4)$$

## 2.5 Engine consumption and carbon emissions

In this paper, the comprehensive fuel consumption model (CMEM) is adopted to calculate the fuel consumption of vehicle [2], which is composed of three modules: engine, speed and load capacity. According to this model, the calculation formula of fuel consumption of vehicles on the road section is shown in (5).

$$F_{ij} = \lambda \left[ KNV + \gamma \beta v_{ij}^3 + \gamma \alpha (\mu + f) v_{ij} \right] \frac{d_{ij}}{v_{ij}} \quad (5)$$

Where:  $\lambda = \xi / \kappa \psi$ ,  $\gamma = 1/1000 \eta_{tf} \eta$ ,  $\alpha = a + g \sin \theta + g C_r \cos \theta$ ,  $\beta = 0.5 C_d \rho A$ ,  $K$  represents the coefficient of engine friction,  $N$  represents the engine speed,  $V$  represents the engine displacement,  $\rho$  represents the air density,  $A$  represents the front surface area of the vehicle,  $C_d$  represents the coefficient of air resistance,  $a$  represents the acceleration,  $g$  represents the gravitational constant,  $\theta$  is the road bend,  $C_r$  is the coefficient of rolling resistance,  $\xi$  represents the mass ratio of air to fuel,  $\kappa$  is the calorific value of the fuel,  $\psi$  represents the fuel conversion factor,  $\eta_{tf}$  represents vehicle rotation efficiency,  $\eta$  represents the parameter of engine efficiency,  $\mu$  and  $f$  represents the self-weight and load capacity of the vehicle, respectively.

$$f_1 = \sum_{(i,j) \in S} \sum_{m \in T} \sum_{k \in K} \lambda x_{ijk} \left( \delta_1 \frac{d_{ij}}{v_{ij}^m} y_{ijk}^{mm} + \delta_2 d_{ij} (v_{ij}^m)^2 y_{ijk}^{mm} + \delta_3 (\mu + f_{ijk}) d_{ij} y_{ijk}^{mm} \right) \quad (6)$$

If there is a situation of driving across time periods, that is, the arrival time of the vehicle from  $i$  to  $j$  is within the  $n$  time zone, then the fuel consumption of all vehicles across the time period can be expressed as:

$$f_2 = \sum_{(i,j) \in S} \sum_{m \in \{T \setminus R\}} \sum_{n \in T, n > m} \sum_{k \in K} \lambda x_{ijk} \left( \delta_1 T_{ij}^{mn} + \delta_2 (t_m - \tau_{ij}) (v_{ij}^m)^3 y_{ijk}^{mn} + \delta_2 \sum_{r=m+1}^{r < n} l_r (v_{ij}^r)^3 y_{ijk}^{mn} + \delta_2 (\tau_{ij} + T_{ij}^{mn} - t_{n-1}) (v_{ij}^n)^3 y_{ijk}^{mn} + \delta_3 (\mu + f_{ijk}) d_{ij} y_{ijk}^{mn} \right) \quad (7)$$

The carbon emission of vehicles is proportional to fuel consumption, and  $m$  is set as the carbon emission conversion factor. The calculation formula of carbon emission is shown in (8).

$$ZC = FC \cdot m = (f_1 + f_2) \cdot m \quad (8)$$

## 2.6 Model construction

Based on the economic and social benefits of integrated distribution enterprises, the objective function is to minimize the sum of vehicle fuel consumption cost, carbon emission cost and dispatch cost.

$$\min C = (f_1 + f_2) \cdot c_f + (ZC - AE) \cdot c_e + \sum_{j \in N_c} \sum_{k \in K} x_{ojk} C_d \quad (9)$$

ST:

$$\sum_{k \in K} \sum_{i, j \in N} x_{ijk} = 1 \quad (10)$$

$$x_{ijk} + x_{jik} \leq 1, \forall (i, j) \in S, k \in K \quad (11)$$

$$\sum_{j \in N_c, i \neq j} f_{jik} - \sum_{j \in N_c, i \neq j} f_{ijk} = q_i x_{ijk}, \forall i \in N_c, k \in K \quad (12)$$

$$f_{i0k} = 0, \forall i \in N_c, k \in K \quad (13)$$

$$f_{ijk} \geq 0, \forall (i, j) \in S, k \in K \quad (14)$$

$$x_{0ik} = x_{j0k}, \forall i, j \in N_c, k \in K \quad (15)$$

$$\sum_{j \in N_c} x_{0jk} \leq k \quad (16)$$

$$q_j x_{ijk} \leq f_{ijk} \leq (Q - q_i) x_{ijk}, \forall (i, j) \in S, k \in K \quad (17)$$

$$\tau_{ijk} = e_{pik} + s_{ik}, \forall p, i, j \in N, k \in K \quad (18)$$

$$e_{ijk} = \tau_{ijk} + T_{ijk}^{mn}, \forall (i, j) \in S, m, n \in \{T \mid n \geq m\} \quad (19)$$

$$T_{ijk}^{mn} = (t_{n-1} - \tau_{ijk}) y_{ijk}^{mn} + \left[ d_{ij} - (t_m - \tau_{ijk}) v_{ij}^m y_{ijk}^{mn} - \sum_{r=m+1}^{r=n-1} l_r v_{ij}^r y_{ijk}^{mn} \right] / v_{ij}^n$$

$$\forall i, j \in N_c, k \in K, m, n \in \{T \mid n \geq m\} \quad (20)$$

$$a_j \leq e_{ij} \leq b_j, \forall i \in N, \forall j \in N_c \quad (21)$$

$$t_{m-1} \leq \tau_{ijk} \leq t_m, \forall (i, j) \in S, m \in T \quad (22)$$

$$\sum_{m \in T \setminus \{R\}} u_{ijk}^m + x_{ijk} = \sum_{m \in T} m w_{ijk}^m, \forall (i, j) \in S, k \in K \quad (23)$$

$$\sum_{m \in T \cup \{0\}} w_{ijk}^m = 1, \forall (i, j) \in S, k \in K \quad (24)$$

$$(t_m - \tau_{ijk}) v_{ij}^m + \sum_{r=m+1}^{r=n-1} l_r v_{ij}^r \leq d_{ij} < (t_m - \tau_{ijk}) v_{ij}^m + \sum_{r=m+1}^{r=n} l_r v_{ij}^r, \forall (i, j) \in S, k \in K, m \in T \setminus \{R\} \quad (25)$$

$$\sum_{m \in T \setminus \{R\}} h_{ijk}^{mn} + m w_{ijk}^m = \sum_{m \in T} n h_{ijk}^{mn}, \forall (i, j) \in S, k \in K \quad (26)$$

$$\sum_{n \in T \cup \{R+1\}, n \geq m} h_{ijk}^{mn} = 1, \forall (i, j) \in S, k \in K, m \in T \setminus \{R\} \quad (27)$$

$$x_{ijk} = \{0, 1\}, \forall (i, j) \in S, k \in K \quad (28)$$

$$y_{ijk}^{mn} = \{0, 1\}, \forall (i, j) \in S, k \in K, m, n \in \{T \mid n > m\} \quad (29)$$

$$h_{ijk}^{mn} = \{0, 1\}, \forall (i, j) \in S, k \in K, m, n \in \{T \mid n > m\} \quad (30)$$

$$u_{ijk}^m = \{0, 1\}, \forall (i, j) \in S, k \in K, m, n \in \{T \mid n > m\} \quad (31)$$

$$w_{ijk}^m = \{0, 1\}, \forall (i, j) \in S, k \in K, m \in T \quad (32)$$

### 3. Algorithm design

In this paper, genetic algorithm is used to solve the model. The idea of genetic algorithm is as follows.

#### 3.1 Coding

Chromosome coding adopts real number coding, and non-zero real number represents the distribution point. Assuming that there are 9 distribution points, the operation of inserting 0 is carried out under the constraint conditions of load capacity and time window, and a legal chromosome code is obtained as (0140825903760), indicating that three distribution vehicles provide distribution services for the 9 distribution points, and the driving path is as follows: path 1:0-1-4-0, path 2:0-8-2-5-9-0, path 3:0-3-7-6-0.

#### 3.2 Fitness function

The objective function in this paper is to minimize the total cost of distribution  $C$ , and the lower

the value, the better. Therefore, the reciprocal of the objective function is used to calculate the fitness value  $F$ .

### 3.3 Genetic operator

After the fitness of each chromosome is calculated, the selection operation is carried out by the roulette selection method, and the selection probability of its chromosome is

$$p(i) = \frac{F(i)}{\sum_{j=1}^m F(j)} \quad (33).$$

In the crossover operation, a partial matching crossover method is adopted to carry out the crossover and recombination of the parent chromosome to generate a new daughter chromosome. The adaptive crossover probability of chromosomes is as follows:

$$P_c = \begin{cases} \lambda_1 \frac{(f - f_{avg})}{f_{max} - f_{avg}}, & f > f_{avg} \\ \lambda_2, & f \leq f_{avg} \end{cases} \quad (34)$$

Where,  $f$  is the higher fitness value of two crossed chromosomes,  $f_{avg}$  is the average value of chromosome fitness value in the population, and  $f_{max}$  is the highest fitness value in the population.

With chromosome A:0-1-4-0-8-2-5-0-8-2-5-6-0 and chromosome B: 0-6-2-5-0-7-6-4-0-6-2-5-0 crossing for example,a sub-path of A and B are randomly selected A sub-path (0-8-2-5-0) extracted in chromosome A is placed before chromosome B to form a new one, chromosome B1 A sub-path (0-1-3-9-0) chromosome B is placed before chromosomes A to form a new one, chromosome A1, and eliminate duplicate gene in the A1 and B1 code to decode the operation, as shown in Figure 2,as shown in Figure 2.

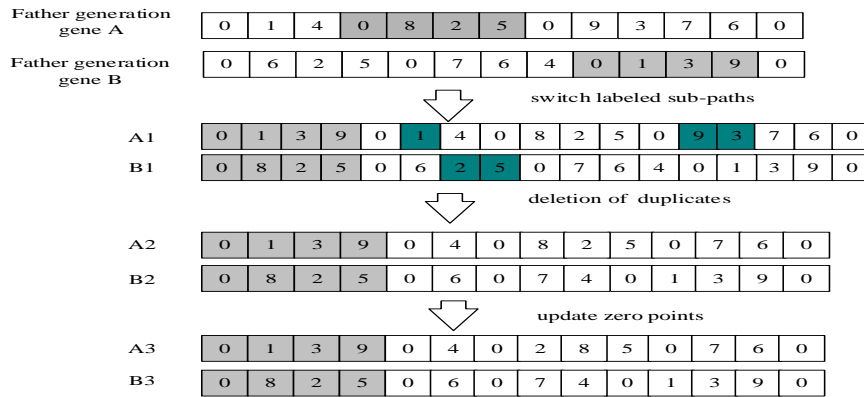


Figure 2 Diagram of Cross Operation

## 4. Conclusions

In this paper, the calculation method of fuel consumption and carbon emission across time periods under time-varying network is presented, and the optimization model of the minimum comprehensive distribution cost is established. The comparison and analysis of the solution results with the objective function of the shortest total mileage and the minimum carbon emission verify the correctness of the model. It provides a distribution plan for logistics and distribution enterprises to achieve both economic and social benefits.

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